PHY 102: DYNAMICS AND ELASTICITY

Chapter:2

Oscillations

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- Displacement, velocity and acceleration in SHM
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Simple Harmonic Motion

Object (eg mass on a spring, pendulum bob) experiences <u>restoring force</u> F, directed towards the equilibrium position, proportional in magnitude to the displacement from equilibrium:

$$F = -kx$$

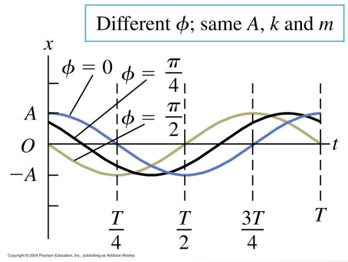
$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega = \sqrt{\frac{k}{m}}$$

Solution: $\omega =$

x(t) =



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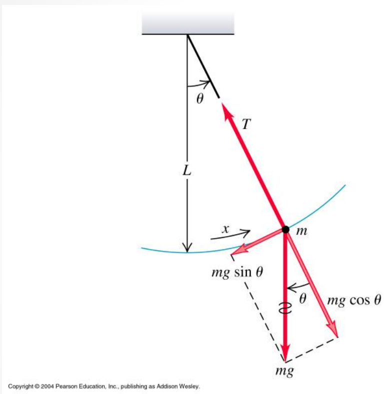
Example: Simple Pendulum (small amplitude)



<u>(NOT SHM for large amplitude)</u>

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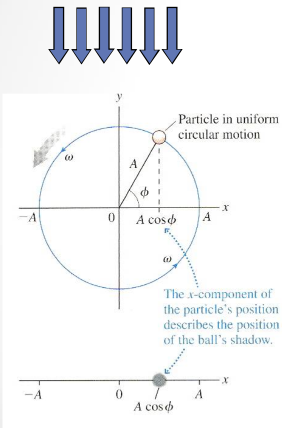
SHM and Circular motion

Light



Displacement of oscillating object = projection on x-axis of object undergoing circular motion

$y(t) = A\cos\theta$

For rotational motion with angular frequency ω, displacement at time t:

$y(t) = A\cos(\omega t + \varphi)$

φ= angular displacement at t=0 (phase constant)
A = amplitude of oscillation (= radius of circle)

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Velocity and acceleration in SHM

$$\text{Displacement}: x(t) = A\cos(\omega t + \varphi)$$

$$\text{Velocity}: v(t) = \frac{dx}{dt} =$$

$$\text{Acceleration}: a(t) = \frac{d^2x}{dt^2} =$$

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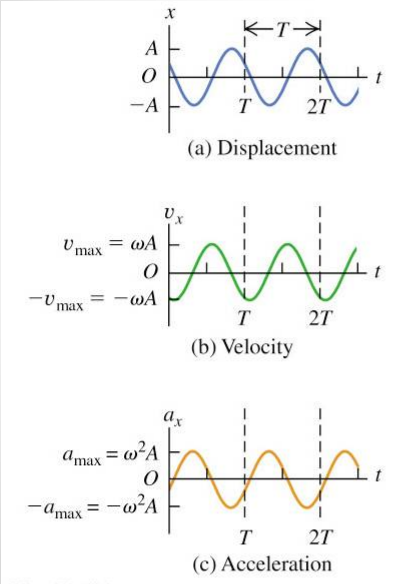
Velocity and acceleration in SHM



$$\text{Displacement}: x(t) = A\cos(\omega t + \varphi)$$

$$\text{Velocity}: v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \varphi)$$

$$\text{Acceleration}: a(t) = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \varphi)$$

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Energy in Simple Harmonic Motion

Potential energy:

Work done to stretch spring by an amount dx = Fdx = -kxdx

Total work done to stretch spring to displacement x:

$$W = \text{stored potential energy} =$$

So, at any time t, the potential energy of the oscillator is given by:

$$PE =$$

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Energy in Simple Harmonic Motion

Kinetic Energy:

At any time t, kinetic energy given by:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 =$$

Total Energy at time t = KE + PE:

$$E =$$

Energy in Simple Harmonic Motion

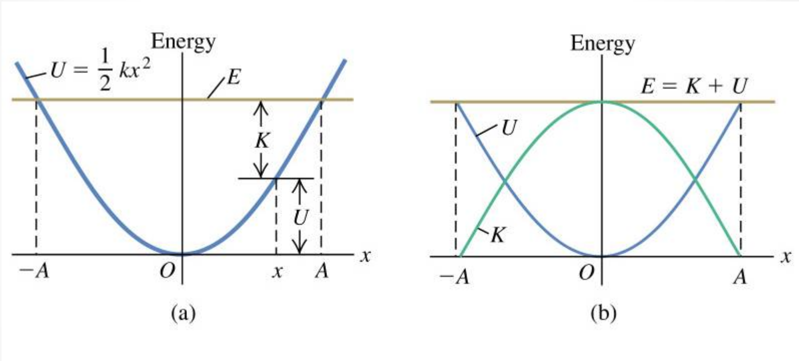
Conclusions

Total energy in SHM is constant

$$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$$

Throughout oscillation, KE continually being transformed into PE and *vice versa, but TOTAL ENERGY remains constant*

Energy in Simple Harmonic Motion



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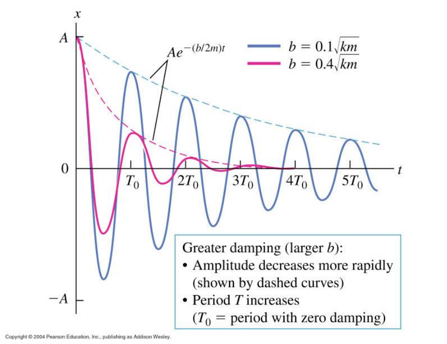
Damped Oscillations

In most “real life” situations, oscillations are always damped (air, fluid resistance etc)

In this case, amplitude of oscillation is not constant, but *decays* with time..(www.scar.utoronto.ca/~pat/fun/JAVA/dho/dho.html)



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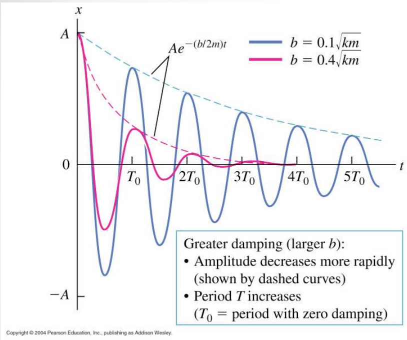
Damped Oscillations



For damped oscillations, simplest case is when the damping force is proportional to the velocity of the oscillating object

In this case, amplitude decays exponentially:

$$x(t) = Ae^{-(b/2m)t}\cos(\omega' t + \varphi)$$

Equation of motion: $$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

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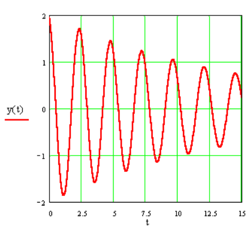
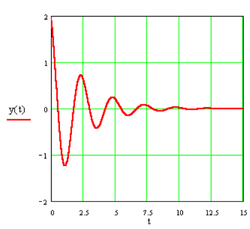
Damped Oscillations

NB: in addition to time dependent amplitude, the damped oscillator also has modified frequency:

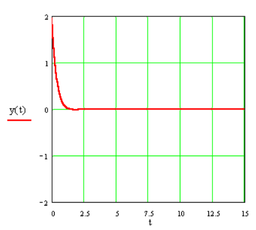
$$\omega' = \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)} = \sqrt{\left(\omega_0{}^2 - \frac{b^2}{4m^2}\right)}$$



Light Damping
(small b/m)



Heavy Damping
(large b/m)



Critical Damping

$$\frac{k}{m} = \frac{b^2}{4m^2}$$

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Forced Oscillations & Resonance

If we apply a periodically varying driving force to an oscillator (rather than just leaving it to vibrate on its own) we have a FORCED OSCILLATION

Free Oscillation with damping:

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

$$\text{Amplitude}=A_0e^{-(b/2m)t}$$

$$\text{Frequency}\quad \omega'=\sqrt{\left(\frac{k}{m}-\frac{b^2}{4m^2}\right)}=\sqrt{\left(\omega_0{}^2-\frac{b^2}{4m^2}\right)}$$

Forced Oscillation with damping:

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=F_0\cos\omega_D t$$

$$\text{Amplitude}=\frac{F_0}{\sqrt{\left(\left(k-m\omega_D{}^2\right)^2+b^2\omega_D{}^2\right)}}$$

MAXIMUM AMPLITUDE WHEN DENOMINATOR MINIMISED:
$k = m\omega_D^2$ ie when driving frequency = natural frequency of the UNDAMPED oscillator

$$\omega_D=\omega_0=\sqrt{\frac{k}{m}}$$

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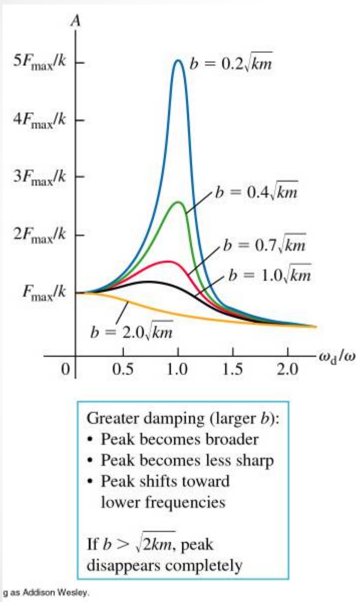
Forced Oscillations & Resonance



When driving frequency = natural frequency of oscillator, amplitude is maximum.

We say the system is in RESONANCE

"Sharpness" of resonance peak described by quality factor (Q)

High Q = sharp resonance

Damping reduces Q

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Thank you
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